Comment on "Peierls Gap in Mesoscopic Ring Threated by a Magnetic Flux"

Yi *et al.* [1] have recently considered the stability of a Charge Density Wave (CDW) in a clean mesoscopic 1D ring pierced by an Aharonov-Bohm flux. Although this letter rises very interesting questions, some results are incorrect or incomplete.

The main result is that a threading flux tends to suppress the Peierls instability, as also claimed in another recent work [2]. This interesting result is only partly correct because it does not properly take into account the parity effect essential in a 1D ring: the thermodynamics depends crucially on the parity of the number N_e of electrons (forgetting the spin) [3,4].

The stability of the CDW is studied through the calculation of the polarization function χ which, in a finite system, is a discrete sum where the wave vector and the nesting vector can take only quantized values. To account for the finite size, the nesting vector is indeed quantized in ref. [1] but the sum is calculated as an integral which does not exhibit the parity effect. The sum (4) of ref. [1] can be indeed calculated exactly. At the best nesting vector $q = 2k_F$, one gets, using the same notations as in ref. [1]:

$$\chi_{2k_F} = \frac{mR}{N_e \hbar^2 k_F} \left(\psi(2k_F R) - \frac{1}{2}[\psi(|f|) + \psi(1-|f|)] \right)$$

for an even number N_e of electrons and $-1/2 < f < 1/2$. f is the dimensionless flux ϕ/ϕ_0. ψ is the digamma function. χ_{2k_F} does not vary logarithmically when $f \to 0$, as claimed in ref. [1], but as a power law. The critical flux f_c does not vary linearly with the size as claimed in ref. [1].

More important, the limits of the discrete sum depend on the parity, as mentioned in the footnote [11] of ref. [1]. Performing the same summation when N_e is odd gives a similar expression for χ_{2k_F} as above where f is changed into $f - 1/2$. Consequently, for a small ring, the Peierls phase does not exist for zero flux and it is *stabilized* above the critical flux $f'_c = 1/2 - f_c$.

More generally, the variation of the order parameter Δ is given by a general equation of the form $\chi(\Delta, T) = 1/Cte$ where the constant is proportional to the interaction parameter. The generalized polarization function $\chi(\Delta, T)$ has the structure of a discrete sum $\sum_n F(n+f)$ which is periodic. The limits of the sum depend on the parity. Using the Poisson summation formula, this sum can be replaced by an integral plus an harmonic expansion in f: $\chi = \int_{-\infty}^{\infty} F(y)dy + 2\sum_{m>0} G_m \cos 2\pi m f$ where G_m has the sign of $(-1)^{N_e m}$. One immediately deduces that changing the parity is equivalent to a shift of the flux by half a period. The complete dependence of the critical temperature and of the gap with the flux and the size are calculated in ref. [4].

Two other comments are of importance concerning the persistent current. First, it is true that the current is *a priori* weak in the Peierls phase and recovers its metallic value above the critical flux f_c (for even N_e). However, the CDW gap vanishes continuously at the critical flux (as agreed by the authors on their fig.1), so that the current cannot be discontinuous at f_c. Indeed the current, very weak at small flux, increases with the flux due to the decrease of the gap and varies *continuously* at f_c. It is found to vary almost linearly below f_c [4] (fig.1a). Secondly, The current shown on the fig.2b of ref. [1] does not present the correct parity effect: the slope has to be always negative in the normal phase, whatever the parity [3]. Indeed, when N_e is odd, the CDW does not exist for small rings and it is *restored* above the critical flux $f'_c = 1/2 - f_c$. Thus, the current for both parities are simply shifted by half a period (fig.1b).

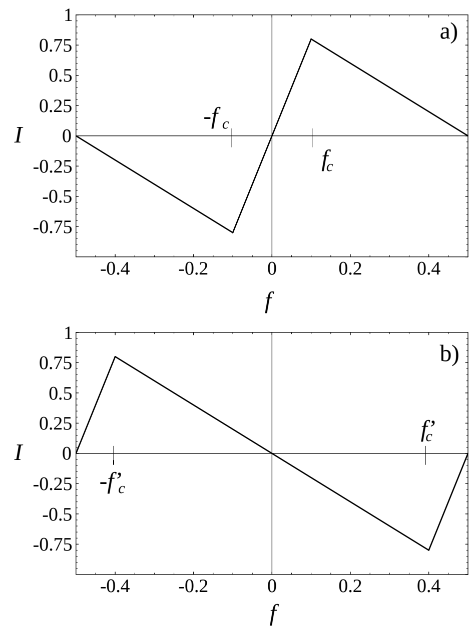

FIG. 1. Persistent current (a) for even N_e, (b) for odd N_e.

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[4] G. Montambaux, Eur. Phys. J. **1**, 377 (1998); similar conclusions have been also found in B. Nathanson *et al.*, Phys. Rev. B **45**, 3499 (1992)